



EFFECTS OF INTERNAL HEAT GENERATION AND THERMAL RADIATION ON UNSTEADY HYDROMAGNETIC FLOW PAST A STRETCHING SURFACE EMBEDDED IN A POROUS MEDIUM WITH VISCOUS DISSIPATION



S. A. Odunlami* and O. J. Ramonu

Department of Mechanical Engineering, Federal Polytechnic, Ilaro, Ogun State, Nigeria

*Corresponding author: odunlamisamson@gmail.com

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Abstract: This study investigates the effects of internal heat generation, thermal radiation and viscous dissipation on an unsteady hydromagnetic boundary layer flow of a viscous incompressible fluid past a stretching surface embedded in a porous medium. Using the similarity transformation, the governing time dependent boundary layer equations are transformed to ordinary differential equations, before being solved numerically using a fourth order Runge-Kutta method. The solutions of non-linear differential equations are obtained and the influence of the Permeability of porosity medium parameter, Heat generation parameter, Unsteady parameter, Magnetic parameter, Radiation parameter, Eckert number and Prandtl number on the velocity and temperature profiles are presented graphically and discussed. It was observed that the temperature distribution in the fluid increases with an increase in the value of the permeability of porosity medium parameter while the velocity decreases. Increasing the internal heat generation parameter produced an increase in the temperature distribution of the fluid in the boundary layer. A comparison with previously published literatures for special cases of the problem show excellent agreement.

Keywords: Boundary layer flow, heat generation, magnetic, radiation, stretching surface

Introduction

Magnetohydrodynamic (MHD) boundary layer flows and heat transfer through a porous medium has generated more interest for its applications in a broad spectrum of processes covering engineering to geophysics such as thermal insulation drying of porous solids, underground disposal of nuclear waste, solid matrix heat exchanges, enhanced oil recovery, cooling of nuclear reactors, food processing and other practical applications. The possible use of porous media adjacent to surfaces to enhance heat transfer characteristics have led to extensive research in heat transfer and flows over bodies embedded in porous media. The problem of flow past a stretching sheet embedded in a porous medium arises in some modern metallurgical and metal-working process involving the cooling of continuous strips or filaments by drawing them through quiescent fluid. The rate of cooling can be controlled and final product of desired characteristics can be achieved if the strips are drawn through porous media.

Sakiadis (1961) initiated the study of the boundary layer flow over a stretched surface moving with constant velocity and formulated boundary layer equations for two-dimensional and axisymmetric flows. Crane (1970) was the first to consider the boundary layer flow caused by a stretching sheet which moves with a velocity varying linearly with the distance from a fixed point. Carragher and Crane (1982) later investigated the heat transfer aspect of the problem under the conditions when the temperature difference between the wall and the ambient fluid is proportional to a power of the distance from a fixed point. The flow and heat transfer can be unsteady due to sudden stretching of the flat sheet or by a step change of the temperature of the sheet (2009). The stream velocity is time dependent and causes unsteadiness in the flow and temperature fields. Recently, Falana and Ramonu (2016) studied the effect of thermal radiation and viscous dissipation on an unsteady MHD boundary layer flow due to stretching sheet and heat transfer, while the effects of variable viscosity and thermal conductivity on an unsteady two-dimensional laminar flow of viscous incompressible conducting fluid past a semi-infinite vertical porous moving plate has been studied by Seddeek and Salama (2007) taking into account the effect of a magnetic field in the presence of variable suction

Abel *et al.* (2007) studied hydromagnetic boundary layer flow and heat transfer in visco-elastic fluid over a continuously

moving permeable stretching surface with non-uniform heat source/sink embedded in fluid-saturated porous medium. Liu (2005, 2006) investigated the heat and mass transfer problems for a viscous fluid-saturated porous medium over an impermeable/permeable and non-isothermal stretching sheet under different environments, respectively.

The study of heat generation effect on the boundary layer flow over a stretching surface is important in problems dealing with chemical reactions and dissociating fluids. The result of possible heat generation might alter the temperature distribution, hence, the particle deposition rate in, semiconductor wafers, electronic chips and nuclear reactors. Vajravelu and Hahjinalaous (1993) investigated the heat transfer characteristic in the laminar boundary layer of a viscous fluid over a stretching surface with viscous dissipation in the presence of internal heat generation or absorption. Samad (2009) studied the effects of mass transfer and radiation on MHD free convection flow along a stretching sheet in presence of heat generation. Heat generation effect on MHD convective flow of a micropolar fluid past a moving vertical porous plate was carried out by Rahman and Sattar (2006).

Mukhopadhyay (2009) studied the effects of variable viscosity on the boundary layer flow and heat transfer of the fluid flow through a porous medium towards a stretching sheet in the presence of heat generation or absorption. Makinde and Sibanda (2011) investigated the chemical reaction effects over the stretching surface in the presence of internal heat generation while Mukhopadhyay and Gorla (2007) derived the problem on heat transfer analysis for fluid flow over an exponentially stretching porous sheet with surface heat flux in porous medium.

In addition, many works have been reported on flow and heat transfer over a stretched surface in the presence of thermal radiation, for some industrial applications such as glass production, furnace design, and in space technology applications such as cosmical flight, propulsion systems, plasma physics aerodynamics rocket and spacecraft re-entry aerothermodynamics which operate at higher temperature, radiation effects are significant. An analytical solution of MHD flow with radiation over a stretching sheet embedded in a porous medium was given by Anjali and Kayalvizhi (2010). Recently, Mahmood (2007) examined the radiation effects on

MHD flow over a stretching surface with variable thermal conductivity. Abd-El-Aziz (2007) studied the effect of thermal radiation and combined heat and mass transfer on hydromagnetic flow over a permeable stretching surface.

Taking into consideration viscous dissipation effects in the study of heat and mass transfer boundary layer problems adds new dimension to it. Parthaet *et al.* (2005) studied the effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface while Liu and Anderson (2008) comprehensively observed the thermal characteristics of a viscous film on an unsteady stretching surface; they concluded that the effect of increasing the unsteadiness is to decrease the rate of heat transfer at the surface. Copiello and Fabbri (2008) also investigated the effect of viscous dissipation on the heat transfer in sinusoidal profile finned dissipaters while Cortell (2005) studied the effects of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet.

Seth *et al.* (2011) studied unsteady MHD Couette flow of a viscous incompressible electrically conducting fluid, in the presence of a transverse magnetic field, between two parallel porous plates. They concluded that velocity increased with increase in Magnetic parameter throughout the channel while Mayongeet *et al.* (2013) investigated the steady MHD Poiseuille flow between two infinite parallel porous plates in an inclined magnetic field. The conclusion was that the velocity was influenced by magnetic inclination, suction/injection rates and the pressure gradient. Devi *et al.* (2014) studied the effect of radiation on MHD free convective boundary layer flow over a stretching sheet in the presence of radiation.

Srivas and Kishan (2015) presented the effects of thermal radiation and chemical reaction on an unsteady MHD flow and heat transfer of nanofluid over a permeable shrinking sheet while Ahmad *et al.* (2015) studied MHD flow and heat transfer through a porous medium over a shrinking/stretching surface with suction. They concluded that velocity profile over a stretching surface decreased with increasing values of magnetic parameter and suction parameter, but reverse effects were noticed for flow over shrinking surface. Ibrahim and Suneetha (2015) investigated the effects of heat generation and thermal radiation on steady MHD flow near a stagnation point on a stretching sheet in porous medium and presence of variable thermal conductivity and mass transfer; they concluded that there is an increase in the temperature profile as the radiation parameter increases and recently, Anyaliet *et al.* (2017) studied the effects of viscous and Joule dissipation on heat transfer in MHD Poiseuille flow in the presence of radial magnetic field. They concluded that increase in Hartmann number has a marked effect on the temperature distribution for cases of Brinkman number which is a case of heat generation due to viscous and Joule dissipation.

In view of the above studies, the present study investigates the effects of internal heat generation, viscous dissipation and thermal radiation on an unsteady hydromagnetic flow past a stretching surface embedded in a porous medium. The governing partial differential equations are transformed in to a system of ordinary differential equation using similarity transformation approach and the resulting equations are then solved numerically using Runge-Kutta shooting method.

Mathematical Model

Consider the unsteady laminar two-dimensional flow of a viscous incompressible electrically conducting fluid past a stretching surface embedded in a porous medium. Fluid is considered in the influence of transverse magnetic field of constant strength B normal to the sheet. A transverse magnetic field of constant strength B , is considered acting normal to the sheet and the induced magnetic field is neglected which is justified for MHD flow at small magnetic Reynolds number.

The Cartesian coordinate system has its origin located at the leading edge of the sheet with positive x -axis extending along the sheet in the direction of flow, while y -axis is along normal to the surface of the sheet and it is positive in the direction from the sheet to the fluid. We assume that for time $t < 0$ the fluid and heat flows are steady. The unsteady fluid and heat flows start at $t = 0$, the sheet is being stretched with the velocity $U_w(x, t)$ along the x -axis, keeping the origin fixed and temperature $T_w(x, t)$. The thermo-physical properties of the sheet and ambient fluid are assumed constant. Under the usual boundary layer approximations, the flow and heat transfer with internal heat generation, thermal radiation and viscous dissipation effects are governed by the following equations.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho} u - \frac{\nu}{k^*} u. \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left(\frac{k}{\rho c_p} \right) \frac{\partial^2 T}{\partial y^2} + \left(\frac{\mu}{\rho c_p} \right) \left(\frac{\partial u}{\partial y} \right)^2 - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{\sigma B^2}{\rho c_p} u^2 + \frac{Q^*}{\rho c_p} (T - T_\infty). \quad (3)$$

Where u and v are the velocity components in the x and y directions respectively, u_∞ is the free stream velocity, μ is the viscosity, T is the temperature of the fluid, K is thermal conductivity of the fluid under consideration, $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, ρ is the fluid density, c_p is the specific heat at constant temperature and q_r is the radiative heat flux. The associated boundary conditions for the velocity and temperature components are given by:

$$y = 0: \quad u = U_w(x, t), \quad T = T_w(x, t). \quad (4a)$$

$$y \rightarrow \infty: \quad u_\infty = 0, \quad T = T_\infty \quad (4b)$$

The stretching velocity $U_w(x, t)$ is assumed to be $U_w = \frac{cx}{(1-\tau t)}$ where both c and τ are both positive constant. We have c as the initial stretching rate $\frac{cx}{(1-\tau t)}$ and it is increasing with time.

We assume the surface temperature $T_w(x, t)$ of the stretching sheet to vary with the distance x along the sheet and time in the following form

$$T_w(x, t) = T_\infty + \frac{c}{2\nu x^2} (1 - \tau t)^{-\frac{3}{2}}. \quad (5)$$

The radiative heat flux q_r is simplified by using Rosseland approximation, Rosseland (1936) as:

$$q_r = -\frac{4\sigma}{3K_e} \frac{\partial T^4}{\partial y}. \quad (6)$$

Where σ is the Stefan- Boltzmann constant and K_e is the Rosseland mean absorption coefficient.

This approximation is valid at points optically far from the boundary surface and it is good for intensive absorption, which is for an optically thick boundary layer. If the temperature differences within the flow are sufficiently small, such that T^4 may be expressed as a linear function of temperature. Expanding T^4 into the Taylor series about T_∞

And neglecting higher order terms yield

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \quad (7)$$

Substituting Eq. (6) and (7) in Eq. (3) gives;

$$\rho c_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \left(k + \frac{16\sigma T_\infty^3}{3K_e} \right) \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \sigma B^2 u^2 + Q^* (T - T_\infty) \quad (8)$$

We introduce the stream function, such that

$$u = \frac{d\psi}{dy}; v = -\frac{d\psi}{dx}. \tag{9}$$

Then, the continuity equation (1) is satisfied;

In order to transform the momentum equation (2) and the energy equation (8) in to set of ordinary differential equation using dimensionless functions f and θ , and the similarity variable η :

$$\eta = y\sqrt{\frac{c}{v(1-\tau t)}} \quad \psi(x, y) = \left(\sqrt{\frac{c}{(1-\tau t)v}}\right) x f(\eta), \quad \theta = \frac{T-T_\infty}{(T_w-T_\infty)}. \tag{10}$$

The velocity components are then derived from the stream function expression and obtained as:

$$u = \frac{Cx}{(1-\tau t)} f'(\eta), \quad v = -\sqrt{\frac{vC}{(1-\tau t)}} f(\eta). \tag{11}$$

The momentum equation (2) and energy equation (8), and the boundary conditions are then transformed to:

$$f''' + f f'' - (f')^2 - (K + M)f' - A(f' + \frac{1}{2}\eta f'') = 0 \tag{12}$$

$$(1 + \frac{4}{3}Ra)\theta'' + Pr_r (Ec(f'')^2 + MEc(f')^2 + 2f'\theta + Q\theta + f\theta' - \frac{A}{2}(3\theta + \eta\theta')) = 0 \tag{13}$$

And the corresponding boundary conditions are

$$\text{At } \eta = 0: f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1$$

$$\eta \rightarrow \infty: f' \rightarrow 0, \quad \theta \rightarrow 0, \tag{14}$$

Where prime (') denote the differentiation with respect to η and dimensionless parameters are:

$$K - \text{Permeability of porous medium} = \frac{v(1-\tau t)}{k^*c}$$

$$Q - \text{Heat generation parameter} = \frac{Q^*x}{\rho c_p u_w}$$

$$M - \text{Magnetic parameter defined as } M = \frac{\sigma B^2(1-\tau t)}{\rho c}$$

$$Ra - \text{Radiation parameter} = \frac{4T^3\sigma}{k K_e}$$

$$Pr - \text{Prandtl number} = \frac{\mu c_p}{k}$$

$$A - \text{Unsteadiness parameter} = \frac{\tau}{c}$$

$$Ec - \text{Eckert number} = \frac{u^2 w}{c_p(T_w - T_\infty)}$$

And the rate of heat transfer (reduced Nusselt number N_u) is transformed as shown:

$$Nu_x = -\frac{x(\frac{\partial u}{\partial y})_{y=0}}{(T_w - T_\infty)} = -\sqrt{\frac{u_w x}{v}} \theta'(0)$$

$$Nu_x = -\sqrt{Re_x} \theta'(0) \tag{15}$$

$$Re_x = \frac{u_w x}{v} \quad (\text{Reynolds number}) \tag{16}$$

Numerical Method for Solution

The set of coupled non-linear governing boundary layer equation together with boundary conditions are solved numerically by using Runge-Kutta fourth order technique along with shooting method. The higher order non-linear differential equations are converted into first order simultaneous linear differential equations which are further transformed into initial value problem by applying the shooting technique and the resultant initial value problem is solved by employing Runge-Kutta fourth order method.

$$f = y_1, \quad f' = y_2, \quad f'' = y_3,$$

$$\theta = y_4, \quad \theta' = y_5,$$

$$f' = y'_1 = y_2,$$

$$f'' = y'_2 = y_3$$

$$f''' = y'_3 \quad \text{and}$$

$$\theta' = y'_4 = y_5$$

Results and Discussion

The boundary layer equations of momentum and energy are solved numerically using Runge -Kutta fourth order algorithm which is implemented in MATLAB as an m-file in the form of ode with $f''(0)$ and $\theta'(0)$ chosen a prior until the boundary conditions at infinity are satisfied. To verify the validity and accuracy of the result obtained, the numerical results for the value of heat transfer rate (reduced Nusselt number) at the wall were compared with Magyari and Keller (1999), Bidin and Nazar(2009), El-Aziz (2010), and Makinde and Seini(2013) for the case of $Q=0, K=0, Ra= 0, M=0$ and steady state case, $A=0$. The present values obtained (Table 1) are in excellent agreement with the previous results.

Table 1: Comparisons of results for reduced Nusselt number $-\theta'(0)$ for various values of Prandtl number Pr
Q=0, K=0, M=0, Ra=0, A=0

Pr	Magyari and Keller (1999)	Bidin and Nazar(2009)	El-Aziz (2010)	Makinde and Seini(2013)	Present study
1.0	0.954782	0.9548	0.9548	0.954811	0.954831
2.0		1.4714		1.471454	1.471460
3.0	1.869075	1.8691	1.869074	1.869069	1.869064
5.0	2.5000132		2.500132	2.500128	2.500082

The effect of Unsteadiness parameter on the velocity profile is shown in Fig 1. It is observed that the velocity also decreases with an increase in the unsteadiness parameter. Fig. 2 is plotted for temperature profiles for different values of the permeability of porosity medium parameter K, it is noticed that the temperature increases with an increase in the value of permeability of porosity medium parameter while Fig. 3 depicts that the velocity profile decreases with an increase of parametric values of the permeability of porosity medium parameter. Fig. 4 shows that increasing internal heat generation parameter produces an increase in the temperature distribution of the fluid in the boundary layer. However, from the results obtained, it is observed that the variations in Internal Heat Generation Q, Prandtl number Pr, Radiation parameter, Ra and Eckert number E do not affect the velocity profile due to the decoupled equation.

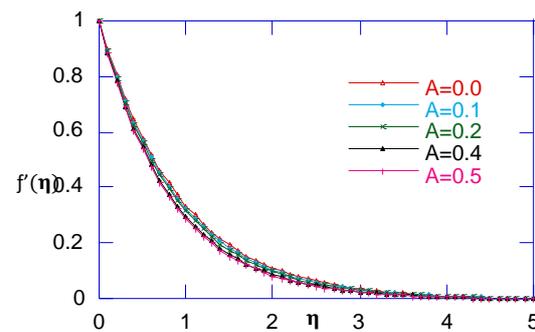


Fig. 1: Velocity profile for various values of unsteadiness parameter when $Ec=0.1, Ra=0.5, Pr=0.7, K=0.1, Q=0.1, M=0$

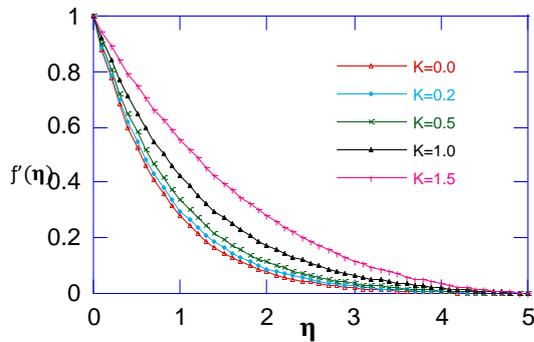


Fig. 2: Velocity profile for various values of permeability parameter when $Ec=0.1$, $Ra=0.5$, $Pr=0.7$, $A=0.8$, $Q=0.1$, $M=0.1$

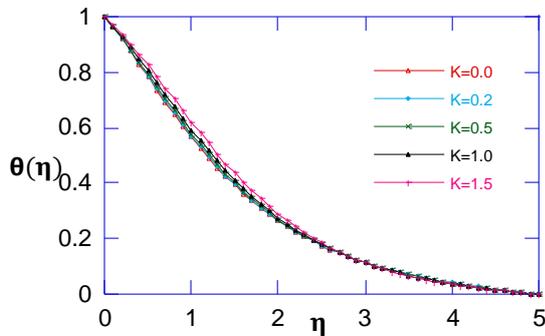


Fig. 3: Temperature profile for various values of permeability parameter when $Ec=0.1$, $Ra=0.5$, $Pr=0.7$, $A=0.8$, $Q=0.1$, $M=0.1$

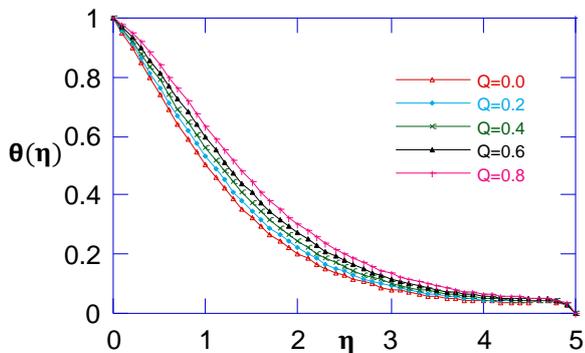


Fig. 4: Temperature profile for various values of internal heat generation parameter when $Ec=0.1$, $Ra=0.5$, $Pr=0.7$, $A=0.8$, $K=0.1$, $M=0.1$

Figure 5 shows that an increase in the thermal radiation parameter produces a significant increase in the thermal boundary thickness and obviously increases the boundary layer temperature distribution. The presence of thermal radiation increases the magnitude of heat transfer rate while Fig. 6 shows the variation of temperature profiles for various values of the Prandtl number. It is observed that, the temperature decreases with increasing values of the Prandtl number in the boundary layer which indicates that temperature in the boundary layer falls very quickly for large values of the Prandtl number. This is because the thickness of the boundary layer decreases with an increase in the value of the Prandtl number.

Figure 7 depicts the velocity profiles for various values of Magnetic parameter, the velocity profiles decrease with increasing Magnetic parameter due to the fact that the magnetic field normal to the flow of an electrically conducting

fluid produces a Lorentz force, and consequently retards the flow. It is observed from Fig. 8 that the temperature profiles for various values of Magnetic parameter, increasing Magnetic parameter shows an increase in the temperature profiles due to the fact that the magnetic field flux slow down the flow at the wall and thereby increases the temperature.

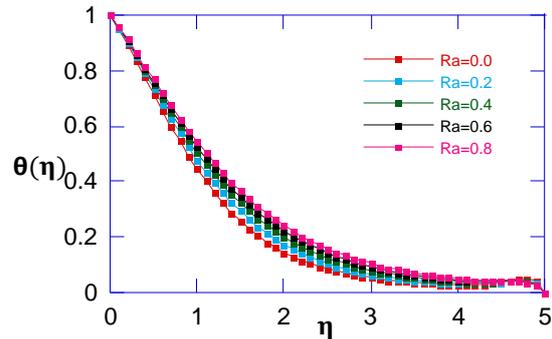


Fig. 5: Temperature profile for various values of radiation parameter when $Ec=0.1$, $Q=0.1$, $Pr=0.7$, $A=0.8$, $K=0.1$, $M=0.1$

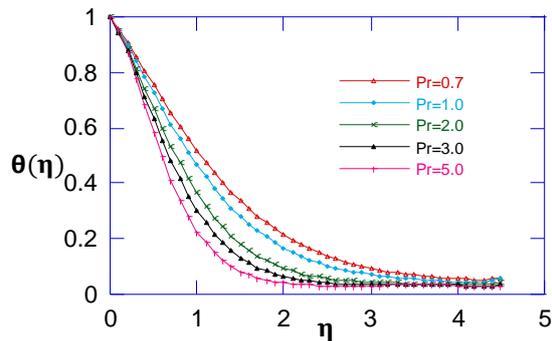


Fig. 6: Temperature profile for various values of Prandtl number when $Ec=0.1$, $Q=0.1$, $Ra=0.5$, $A=0.8$, $K=0.1$, $M=0.1$

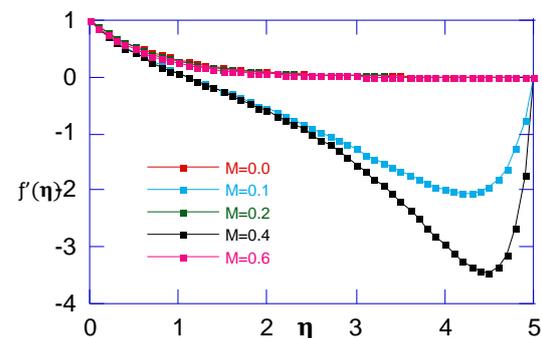


Fig. 7: Velocity profile for various values of magnetic parameter when $Ec=0.1$, $Q=0.1$, $Ra=0.5$, $A=0.8$, $K=0.1$, $Pr=0.7$

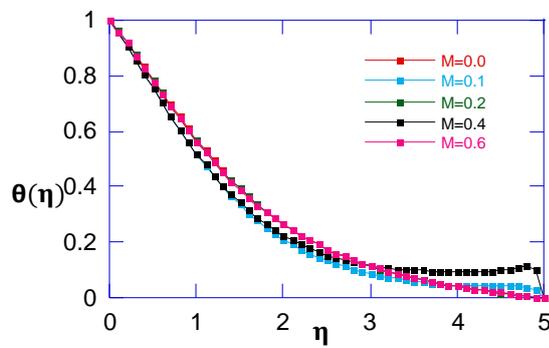


Fig. 8: Temperature profile for various values of magnetic parameter when $Ec=0.1, Q=0.1, Ra=0.5, A=0.8, K=0.1, Pr=0.7$

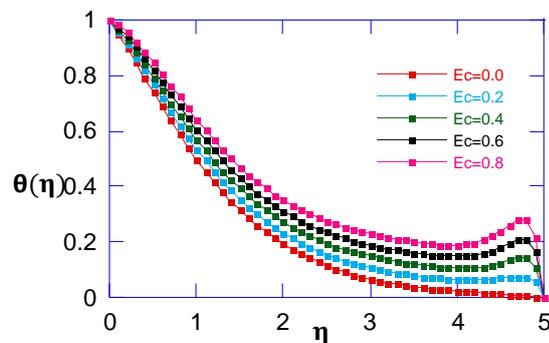


Fig. 9: Temperature profile for various values of Eckert number when $M=0.1, Q=0.1, Ra=0.5, A=0.8, K=0.1, Pr=0.7$

Table 2: Values of reduced Nusselt number $-\theta(\eta)$ and skin friction $-f'(\eta)$ for varying values of Pr, M, K, Q, A and Ra

Parameters	Values	$-\theta(\eta)$	$-f'(\eta)$
Pr	0.7	0.41361	1.42782
	1.0	0.43815	1.42782
	1.5	0.45586	1.42782
	2.0	0.45760	1.42782
M	0.5	0.37253	1.45242
	1.0	0.38365	1.65834
	1.5	0.38114	1.74662
K	0.2	0.38368	1.87737
	0.3	0.41380	1.45614
	0.4	0.37252	1.38661
Q	0.4	0.41289	1.50954
	0.5	0.37744	1.45242
	0.2	0.38103	1.42782
	0.3	0.34722	1.42782
A	0.4	0.31207	1.42782
	0.5	0.27543	1.42782
	0.2	0.08912	1.15860
	0.4	0.10901	1.21595
Ra	0.6	0.25283	1.30901
	0.8	0.41361	1.42782
	1.0	0.38646	1.42782
	1.5	0.36513	1.42782
	2.0	0.34804	1.42782
	3.0	0.32231	1.42782

Figure 9 obviously indicates that the effect of increasing the values of the Eckert number is to increase temperature distribution in the flow region. This behavior of temperature enhancement occurs as heat energy is stored in the fluid due to frictional heating.

As observed in Table 1, the heat transfer coefficient increases with an increase in Prandtl number. This is true because by definition, Prandtl number is the ratio of kinematic viscosity to thermal diffusivity. An increase in the values of Prandtl number implies that momentum diffusivity dominates thermal diffusivity. Hence the rate of heat transfer at the wall increases with increasing values of Prandtl number.

Conclusion

For various values of relevant physical parameters to the study including the magnetic parameter (M), the effects of internal heat generation, thermal radiation and viscous dissipation on an unsteady hydromagnetic boundary layer flow of a viscous incompressible fluid past a stretching surface embedded in a porous medium has been studied. From the present investigation, the following conclusions may be drawn:

- (i) It is observed that the thermal boundary layer thickness is increased in the presence of heat generation
- (ii) The velocity decreases while the temperature increases along the stretching surface with the increase of magnetic parameter.
- (iii) It is noticed that the temperature increases with an increase in the value of permeability of porosity medium parameter while the velocity decreases.
- (iv) It is established that an increase in the thermal radiation parameter produces a significant increase in the thermal boundary thickness and obviously increases the boundary layer temperature distribution.
- (v) The heat transfer coefficient increases with an increase in Prandtl number.

Conflict of Interest

The authors declare that there is no conflict of interest regard whatsoever.

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